Computations of non-linear dynamics of shells using Energy-Momentum methods

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Introduction

Most of the well known implicit algorithms developed for linear dynamics, do not conserve energy and angular momentum in non-linear cases. These algorithms become unstable in long-time simulations of non-linear dynamics. The Energy-Momentum methods provide suitable algorithms that conserve in each time step momentum and total energy in Hamiltonian systems.

- conservation of energy
  
  \[ [\psi_{in} + \psi_{ext} + K]_{n+1} = [\psi_{in} + \psi_{ext} + K]_n \]

  \( \psi_{n+1} \) and \( \psi_n \) are the potential energies at the end and the beginning of the time step. \( K_{n+1} \) and \( K_n \) are the corresponding kinetic energies.

  \[ \psi_{in} = \psi_{in}(C) \]

  \[ C = F^T F. \]

  \[ \psi_{ex} = -\int_B P \cdot x. \]

  \[ K = \int_B \frac{1}{2} \dot{x} \cdot \dot{x} dv. \]

- Conservation of linear momentum in the absence of external forces

  \[ L_{n+1} = L_n \]

  \( L_{n+1} \) and \( L_n \) are the linear momenta at the end and the beginning of the time step.

  \[ L = \int_B \rho \dot{x} dv \]

- conservation of angular momentum in the absence of external

  \[ J_{n+1} = J_n \]

  \( J_{n+1} \) and \( J_n \) are the angular momenta at the end and the beginning of the time step.

  \[ J = \int_B (x - X^0) \times \rho \dot{x} dv \]

Example 1  Tumbling of a plate

A free motion of the plate is simulated using 4 noded elements and a 6 x 6 mesh. The load increases linearly to a maximum and vanishes. The time integration algorithm conserves linear and angular momentum as well as the total energy.

Example 2  Snap-through of a cylindrical shell

A \( \frac{1}{4} \) of the shell is discretized using 4-noded elements and a 4 x 4 mesh. The load increases linearly to a maximum value and remains constant. The diagrams depict snap-through phenomenon simulated using the Energy-Momentum algorithm.