Analysis of a Micropolar Continuum using Length Scales and Imperfections

Diploma Thesis from Jörg-Christian Ebert

Universität Karlsruhe (TH)

BAUSTATIK

Extended micropolar theory

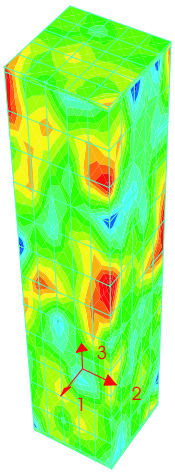
Micropolar theory strain is defined as product of the rotational field $\mathbf{R}$ and the deformation gradient $\mathbf{F}$ in the way $\bar{\mathbf{U}} = \mathbf{R}^{\mathbf{T}}\mathbf{F}$.

Standard formulation for strain energy is extended to

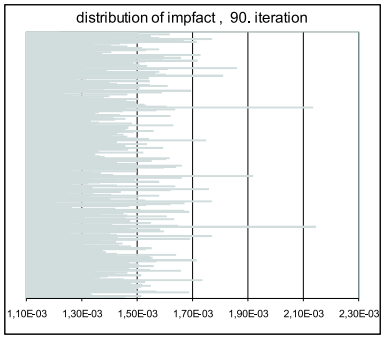
$$\begin{aligned} W_{mp}(\bar{\mathbf{U}}) &= (1-\vartheta)\mu \left\| sym(\bar{\mathbf{U}} - \mathbb{1}) \right\|^2 \\ &+ \mu_c \left\| skew(\bar{\mathbf{U}} - \mathbb{1}) \right\|^2 \\ &+ \frac{\lambda^\star}{4}\left([det\bar{\mathbf{U}} - 1]^2 + [\frac{1}{det\bar{\mathbf{U}}} - 1]^2 \right) \\ &+ \vartheta\mu \left\| sym\left(cof(\bar{\mathbf{U}}) \right) - \mathbb{1} \right\|^2 . \end{aligned}$$

Algorithm for stochastic imperfections

Imperfection field



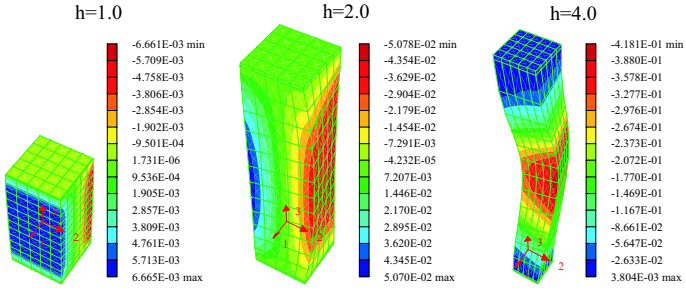
Imperfections are used to avoid critical states of stability and to lead computations on a stable path. We introduce imperfections as a stochastic micro-rotational field $\mathbf{R_{imp}}$.



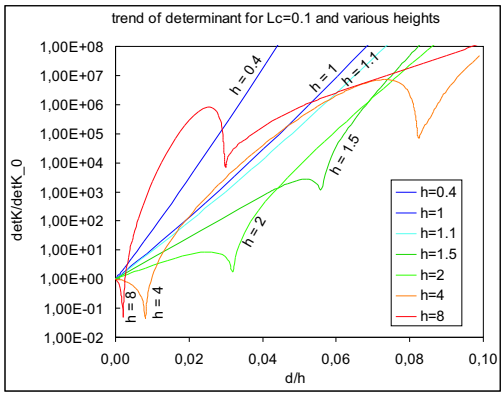
The micro-rotational field $\mathbf{R}$ is disarranged directly through $\bar{\mathbf{R}} = \mathbf{R_{imp}}\,\mathbf{R}$.

Compression test for various heights of structure

For the same cross section we vary the height of structure and slenderness with it. Deformed structure and displacement field in 1-direction:

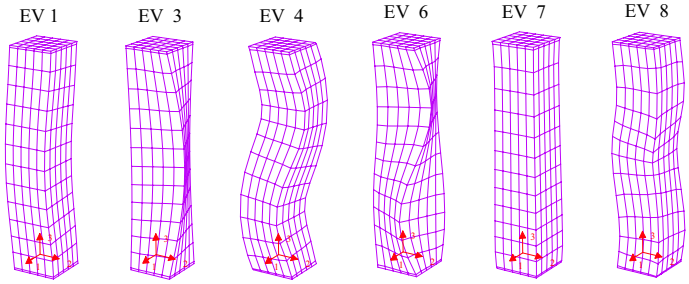


For slender structures we observe buckling in addition to twist around the 3-axis. Both, twist and buckling can be seen in the trend of stiffness matrix determinants as minimal points.

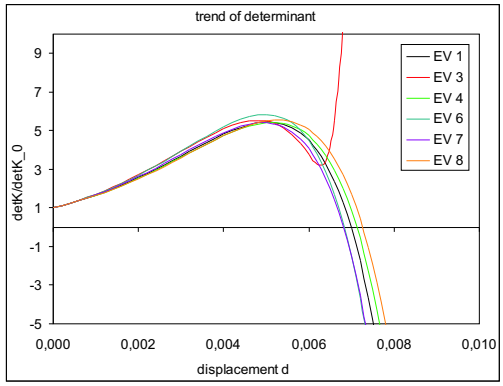


Eigenvectors as imperfections

Eigenvectors out of an eigenwert analysis are superimposed to the undeformed mesh.



Without imperfections the compressed sample would sooner or later reach a point of junction. The indicator of that is the global stiffness matrix determinant approximating zero.



The trend of determinants shows that only the sample with eigenform 3 has the decisive imperfection which also results from a stochastic disturbance of the micro-rotational field.