Karlsruher Institut für Technologie

Institut für Baustatik
Leiter: Prof. Dr.-Ing.
habil. W. Wagner

Mathematical and Statistical Models for the Approximation of Critical Loads in Composite Materials with Fibres-Discontinuities

Eduard Brull i Piqué

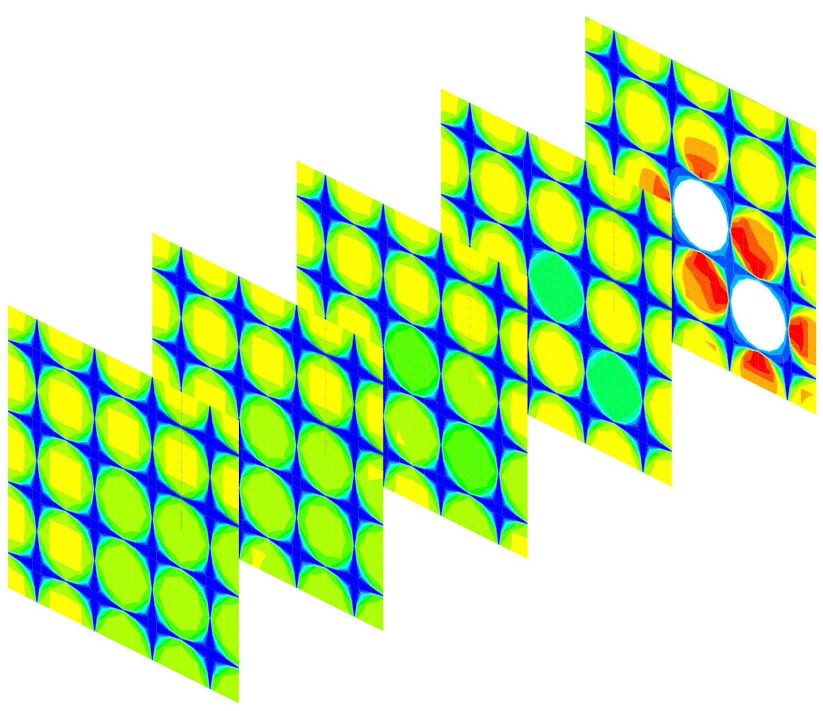
1. Motivation

In the study of critical loads of Unidirectional Carbon Fibre - Reinforced Epoxy Composites, literature reports a huge divergence between the observed results and the predicted-by-the-theory ones. Among other things, that is due to the Stress Concentration Factor (SCF or K) that appears on a fibre when other fibres within the composite present discontinuities or breakage. This factor is defined as the ratio between the observed axial stress on the centre of a fibre and the ideal axial stress that would appear there according to the theory and assuming that no broken fibres exist.

$$K_i = \frac{\sigma^i_{xx}}{\sigma^{ideal}_{xx}}$$

2. Mathematical model

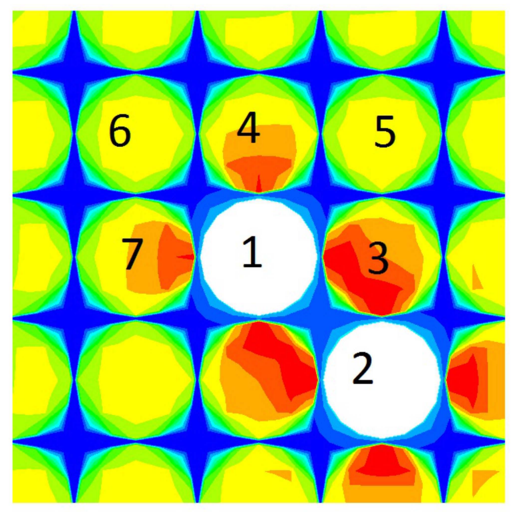
Several simulations with $FEAP$ are carried out to study the relation between the SCF and the characteristics of the composite material (Young's Modulus and Poisson's Ratio of Fibre and Matrix and their relative concentration v). Mathematical relations are found in order to obtain the SCF distribution within a composite with several fibres in it.



$$\sum \epsilon(i) K_i = N$$

with $\epsilon(i) \begin{cases} 0 \text{ if broken fibre } (N_b) \\ 1 \text{ if fibre is intact } (N - N_b) \end{cases} \qquad K_i = \prod_{j=1}^{N_b} K_i^j$

In these equations, N is the total number of fibres of the considered section of study and N_b is the total number of broken fibres in that section.

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K   MATRIX

number of fibers =          25

broken fiber are          3          3
broken fiber are          4          4

1.0538 1.0792 1.0958 1.0902 1.0741

1.0792 1.1192 1.2015 1.1418 1.1070

1.0958 1.2015 0.0000 1.3040 1.1418

1.0902 1.1418 1.3040 0.0000 1.2015

1.0741 1.1070 1.1418 1.2015 1.1192
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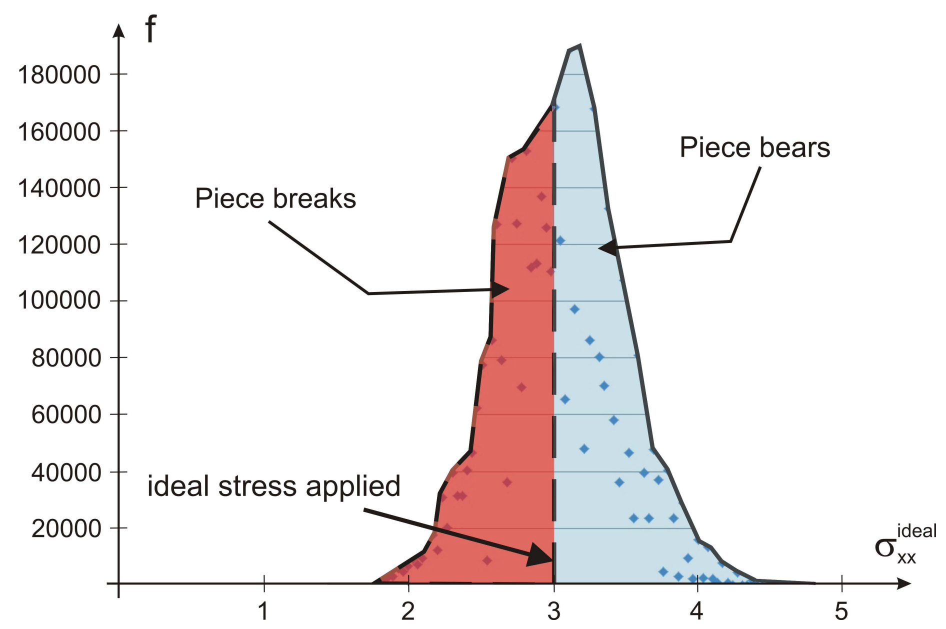
3. Statistical model

In order to obtain a more rigorous and close-to-reality result, the distribution of broken fibres is varied randomly. A threshold of broken fibres (u), probability that a fibre is not broken, is set. Several calculations for each threshold (u) and applied load ($\sigma^{ideal}_{xx} = \frac{F}{A_T v}$, where A_T is the total area of the considered section of study and v is the concentration of fibres within the composite) are carried out for calculating the maximum value of the Stress Concentration Factor (K) and its frequency f. The criterion of rupture of the composite material is set as:

$$\sigma^{ideal}_{xx} = \frac{\sigma_c}{K_{max}}$$



Therefore, the probability that the composite material piece will not resist the applied load is the sum of all the calculated cases where the critical ideal stress is lower than the applied ideal stress.

$$P_{break} = \frac{1}{N} \cdot \sum^{i} f(\sigma^{ideal}_{xx})$$

with $N = \sum f(\sigma^{ideal}_{xx}) \quad i = \sigma^{c}_{xx} < \sigma^{applied}_{xx}$

	p security			
treshold	0.5	0.9	0.95	0.99
0.95	1.871232	1.63024	1.552272	1.318368
0.99	2.431184	2.154752	2.083872	1.935024
0.995	2.55168	2.289424	2.225632	2.083872