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Topology Optimization for Composites with Phase Field Modeling and Isogeometric Analysis

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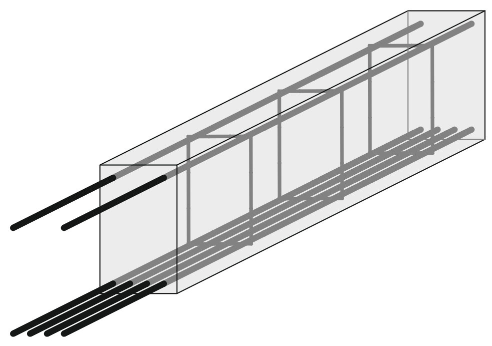
1. Motivation



The load-bearing capacity of a composite can be increased by optimally positioning the used materials within the component. For this purpose, a phase field model is developed which performs topology optimization of a two-component composite. Similar to reinforced concrete, the reinforcing material can hold tensile stresses to a much higher extent than the matrix. Mechanical fields and the phase field are determined by isogeometric analysis (IGA) to obtain a high approximation order.

2. The Phase Field Model

A phase field variable φ is introduced, which indicates regions of matrix material for $\varphi = -1$ and regions of reinforcing material for $\varphi = +1$. A diffuse transition zone interpolates between the two states. Main part of the total energy functional

$$\Pi = \int_\Omega \left[W_{\text{int}} + \Psi_{\text{well}} + \Psi_{\text{grad}} + \Psi_{\text{mech}} + W_{\text{ext}}\right] dV$$

is the internal work contribution

$$W_{\text{int}} = c_\gamma \varphi \, \frac{\bar{\sigma}_1(\hat{K}) - \sigma_1(\varphi, \boldsymbol{\varepsilon}(\mathbf{u}))}{\bar{\sigma}_1(\hat{K})} \, .$$

It reduces inhomogeneity of the first principal stress σ_1 within the design domain. The transition zone at interfaces remains thin due to the double-well potential Ψ_{well}. Large interfaces as well as high gradients of φ are penalized by Ψ_{grad}. The mechanical energy Ψ_{mech} and contributions from external loads W_{ext} are also taken into account. Minimizing the total energy with respect to displacements $\mathbf{u}$ and phase field variable φ leads to an optimized topology. During this process, the phase field variable evolves according to the non-conserving Allen-Cahn equation

$$\frac{\partial \varphi}{\partial t} = -\frac{1}{\omega} \frac{d\Pi}{d\varphi} \, .$$

The volume share of reinforcement is implicitly imposed by $\hat{K}$.

3. Isogeometric Analysis

IGA is used to determine displacements $\mathbf{u}$ and phase field variable φ. It generalizes the finite element method by employing non-uniform rational B-Splines (NURBS) for the approximation. The IGA shape functions in two and three dimensions read

$$R_{i,j}^{p,q}(\xi,\eta) = \frac{N_{i,p}(\xi)\, M_{j,q}(\eta)\, w_{i,j}}{\sum_{i=0}^{n} \sum_{j=0}^{m} N_{i,p}(\xi)\, M_{j,q}(\eta)\, w_{i,j}}$$

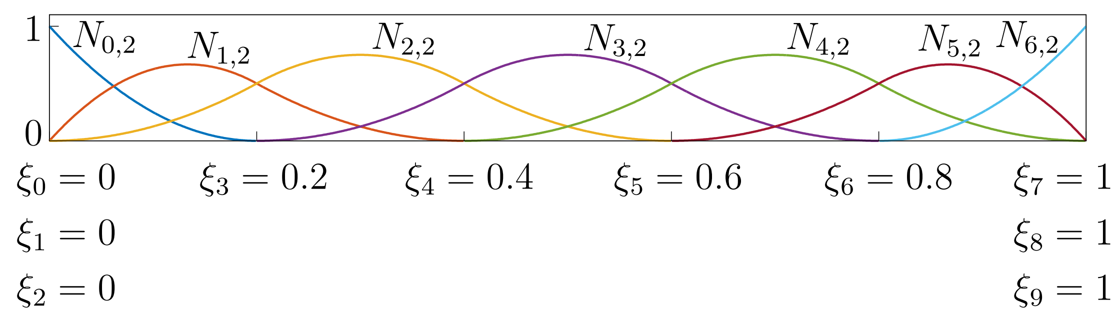
$$R_{i,j,k}^{p,q,r}(\xi,\eta,\zeta) = \frac{N_{i,p}(\xi)\, M_{j,q}(\eta)\, L_{k,r}(\zeta)\, w_{i,j,k}}{\sum_{i=0}^{n} \sum_{j=0}^{m} \sum_{k=0}^{l} N_{i,p}(\xi)\, M_{j,q}(\eta)\, L_{k,r}(\zeta)\, w_{i,j,k}}$$

with weights $w > 0$ and B-spline basis functions

$$N_{i,0}(\xi) = \begin{cases} 1 \text{ if } \xi_i \leq \xi < \xi_{i+1} \\ 0 \text{ otherwise} \end{cases}$$

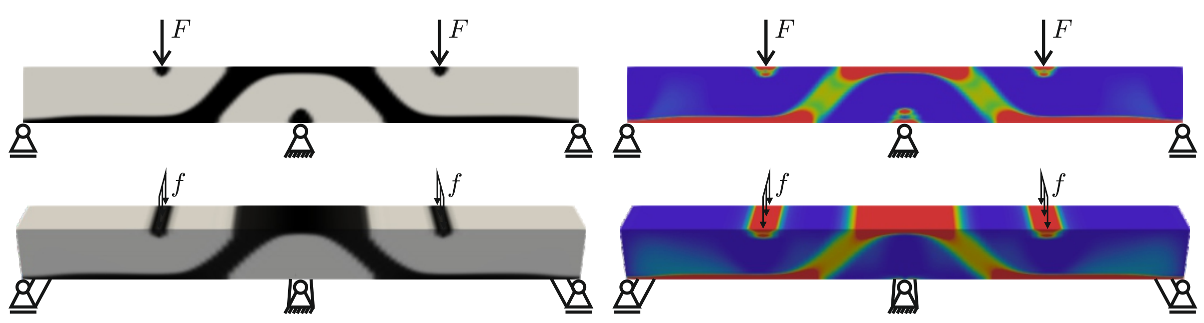
$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi) \, .$$

They are defined on a knot vector $\Xi = \{\xi_0, \xi_1, \ldots, \xi_{n+p+1}\}$, containing monotonously increasing real values.



4. Numerical Examples

The phase field model is applied to a two-dimensional and to a three-dimensional two-span beam of dimensions $L = 20\,m$, $H = 2\,m$, and in three dimensions $W = 2\,m$. The loads induce pressure singularities with high transversal tension stresses. In two dimensions, plane stress is assumed and the beam is discretized with 160×80 integration cells. To reduce the computational cost, simulations of the three-dimensional beam are carried out on one half of the system by taking into account its symmetry. The halved beam is discretized with $52 \times 52 \times 12$ integration cells. In both cases, cubic NURBS shape functions are employed. For $\hat{K} = 0.2$, optimized topologies of the reinforcement are obtained and the corresponding fields of the first principal stress σ_1 are considered:



Reinforcement is positioned in areas of high flexural tensile stress and combined to a coherent structure. In addition, small crescents of reinforcing material account for high transversial tension stresses. The evolving topology concentrates tensile stresses within the reinforcement and thus reduces them within the matrix. For varying mechanical properties and values of $\hat{K}$, the model successfully generates plausible topologies.