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# Topology Optimization of Structures with Stress and Displacement Constraints

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## 1. Motivation

When designing a structure, an adequate geometry has to be chosen, to transfer loads in a safe and also efficient way. The best possible solution can be found by optimizing the geometry of the structure using mathematical methods. Thus, material usage can be reduced, resulting in lower resource consumption and more lightweight designs.

## 2. Problem Formulation

In topology optimization, the material distribution in a design domain is optimized. Therefore, the design domain is discretized into pixels, which have a relative material density ranging between zero (white) and one (black).

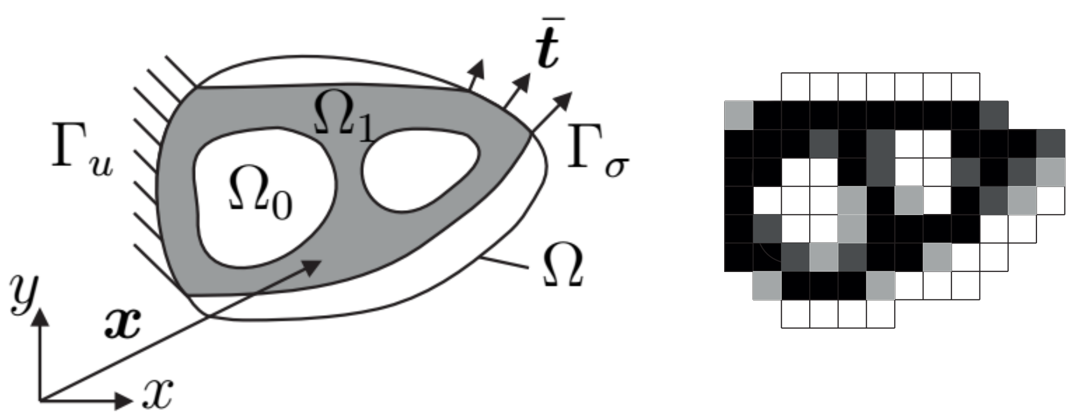


**Figure 1:** *General concept of topology optimization.*

**Optimization Problem** The objective of the optimization is to minimize the volume and thus the amount of material of the structure. This should be achieved while meeting all requirements being specified in the form of stress and displacement constraints. The corresponding optimization problem is written as

$$\begin{aligned}
\min_{\rho} \quad & V_f = \frac{1}{V_0}\sum_{e=1}^{Ne}\bar{\rho}_e V_e \\
\text{s. t.} \quad & \eta_\sigma^{(e)} - 1 = \frac{\sigma_{eq}^{(e)}}{\sigma_{lim}} - 1 \leq 0 \qquad e = 1, ..., N_e \\
& \eta_u^{(n)} - 1 = \frac{u_y^{(n)}}{u_{y,lim}} - 1 \leq 0 \qquad n = 1, ..., N_n \\
& \boldsymbol{K}\,\boldsymbol{U} = \boldsymbol{F} \\
& 0 < \rho_e \leq 1.
\end{aligned}$$

## 3. Solution Strategy

In order to solve the optimization problem, it is treated as an unconstrained problem using the Augmented Lagrangian (AL) method. For purely stress-constrained optimization, the AL function is given as

$$L(\bar{\rho}, \boldsymbol{\lambda}_i, r_i) = V_f(\bar{\rho}) + \frac{r_\sigma}{2}\sum_{e=1}^{N_e}\left(\max(0, \frac{\lambda_\sigma^{(e)}}{r} + \eta_\sigma^{(e)} - 1)\right)^2.$$

Subsequently, the gradient can be derived analytically and the steepest descent method (SDM) is applied to obtain the minimum of the AL function in an iterative manner, such as

$$\rho_i^{(b+1)} = \rho_i^{(b)} - \Psi\,(\nabla_\rho L)_i^{(b)}.$$

## 4. Numerical Examples

**Cantilever Beam** The left boundary is supported and a point load is applied at the top right corner. Elastic support or removal of stress constraints for certain elements can be used to avoid problems with stress concentrations at the support.

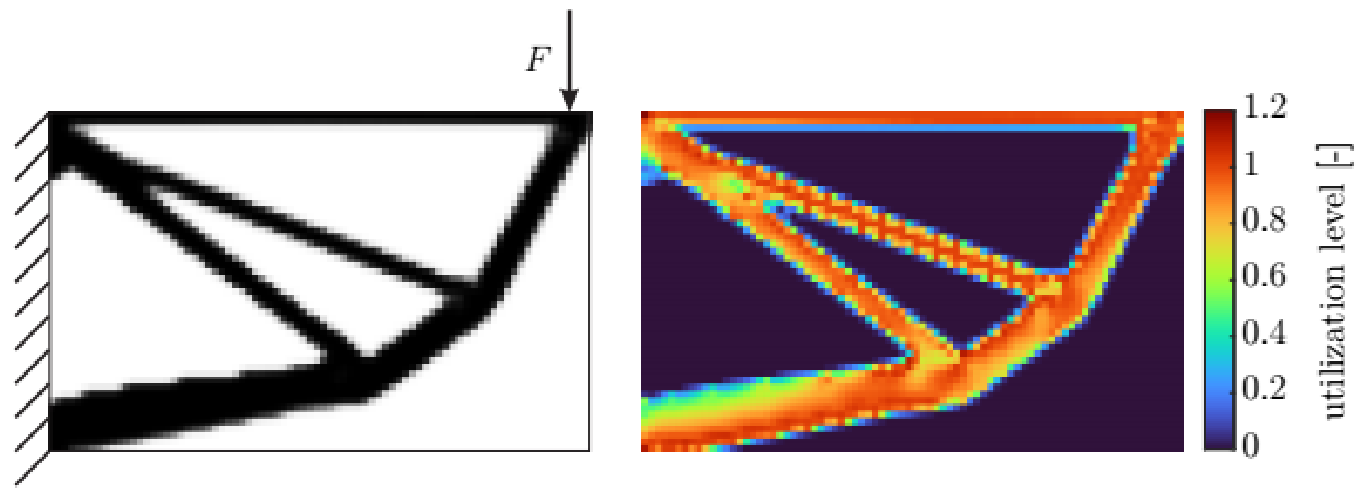


**Figure 2:** *Optimized topology and stress distribution for the cantilever beam with elastic support.*

**Two-Span Beam** A beam with two different span widths is considered. It is subjected to a uniform load $q$ applied at the bottom. Different displacement limits can be specified for each span. At the bottom, two rows of elements are fixed to a density of one.

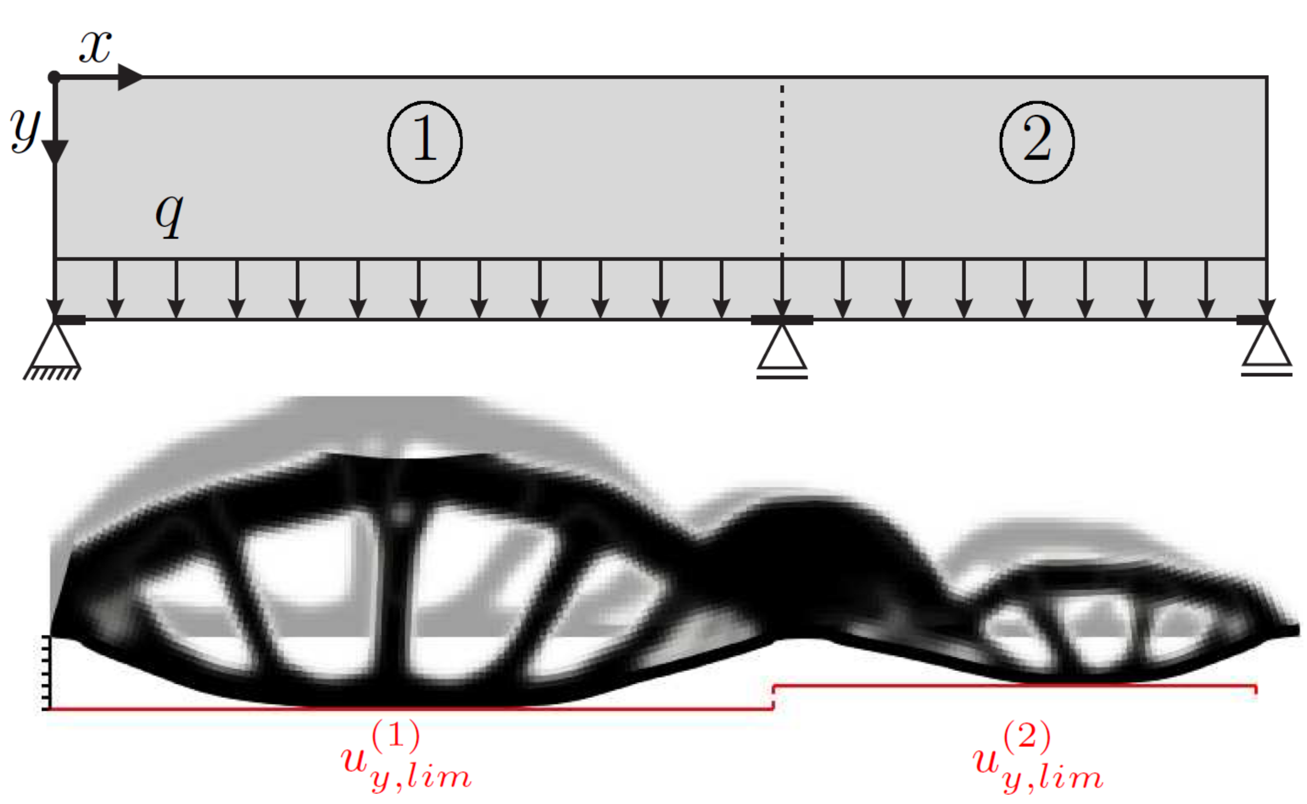


**Figure 3:** *Two-span-beam: (a) Design domain and boundary conditions (b) Optimal topology and displacements (scaling factor 100).*

It is concluded, that formulating customized optimization problems based on specific engineering requirements is a promising concept when applying topology optimization in structural engineering.